Key

Math 1

4-4 Exponential Decay & Half-Life

Name____________________________

Date__________

- *I can define an exponential function.*
- *I can analyze the input and output values of a function based on a problem situation.*
- *I can convert a sequence into a recursive or explicit formula.*
- *I can correctly choose which formula best models a given situation.*
- *I can determine the practical domain and range in the context of a problem. And explain how they are related to the graph.*

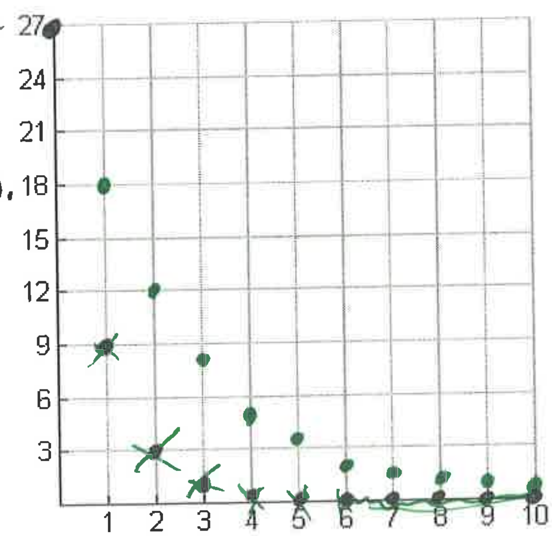
1. Most Popular American sports involve balls of some sort. In designing those balls, one of the most important factors is the bounciness or elasticity of the ball. For example, if a new golf ball is dropped onto a hard surface, it should rebound to about $\frac{2}{3}$ of its drop height.

a. *Fill in the table below, then plot on the groph.*

Bounce #	0	1	2	3	4	5	6	7	8	9	10
Height (ft)	27	18	12	8	5.3	3.5	2.37	1.58	1.05	0.70	0.47

a. How does the rebound height change from one bounce to the next? How is that pattern shown by the shape of the graph?

$\frac{1}{3}$ of the previous height. Multiply by $\frac{1}{3}$.
Graph decreases at a decreasing rate (fast then slow).



b. Write a recursive rule for the data.

$$\begin{cases} a_0 = 27 \\ a_n = a_{n-1} \cdot \frac{2}{3} \end{cases}$$

c. Write an explicit rule for the data,

$h(b) = 27\left(\frac{2}{3}\right)^b$

d. If the ball was dropped from 15 feet instead of 27 feet…

i. How would the graph be different?

Lower – the y-intercept would be 15 instead of 27.

ii. How would the recursive rule be different?

27 becomes 15. Everything else is the same.

iii. How would the explicit rule be different?

Same answer as d ii.

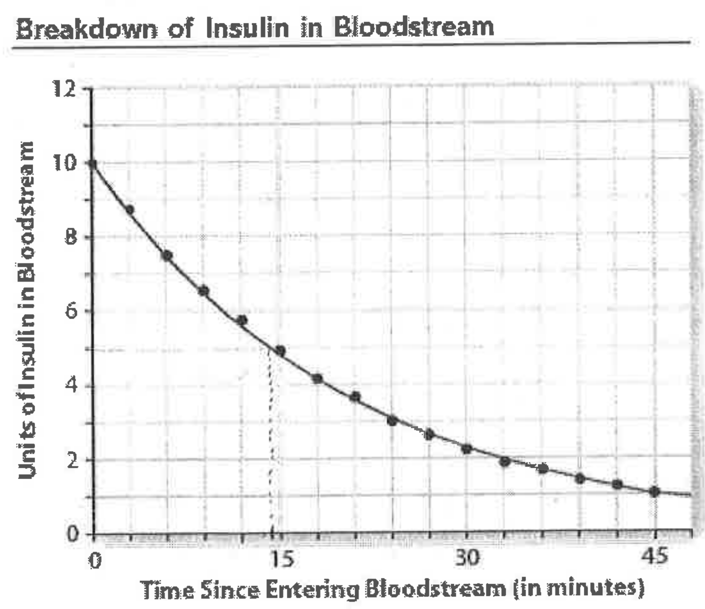
Definition of **half-life**: Amount of time it takes for a substance to decrease to half of the initial amount.

Prescription drugs are a very important part of the human health equation. Many medications are essential in preventing and curing serious physical and mental illnesses.

Diabetes, a disorder in which the body cannot metabolize glucose properly, affects people of all ages. In 2005, there were about 14.6 million diagnosed cases of diabetes in the United States. It was estimated that another 6.2 million cases remained undiagnosed. (Source: diabetes.niddk.nih.gov/dm/pubs/statistics/index.htm)

In 5–10% of the diagnosed cases, the diabetic's body is unable to produce insulin, which is needed to process glucose.

To provide this essential hormone, these diabetics must take injections of a medicine containing insulin. The medications used (called insulin delivery systems) are designed to release insulin slowly. The insulin itself breaks down rather quickly. The rate varies greatly among individuals, but the following graph shows a typical pattern of insulin decrease.



1. Medical scientists often are interested in the time it takes for a drug to be reduced to one half of the original does. They call this time the **half-life** of the drug.

The half-life of insulin appears to be about 15 min.
I know that this is the half-life because. . .

That's when there are $\frac{10}{2} = 5$ units left.

2. The pattern of decay shown on this graph for insulin can be modeled well by the function $f(x) = 10(0.95)^x$, where x is the number of minutes since the of insulin entered the bloodstream.

   a. The number 10 in the function tells me: initial amount of insulin = 10 units

   and the number 0.95 tells me decay rate... 95% of insulin remains after each min.

   b. The percent of insulin that is *used* each minute is 5%. I know this because. . .

   95% remains & 100 − 95 = 5

3. Write a recursive rule that shows how the amount of insulin in the blood changes from one minute to the next.

$$\begin{cases} a_0 = 10 \\ a_n = a_{n-1} \cdot 0.95 \end{cases}$$

Show how to do in calc

4. Mathematicians have figured out ways to do calculations with fractional or decimal exponents so that the results fit into the pattern for whole number exponents. One of those methods is built into your graphing calculator. Enter the function $f(x)=10(0.95)^x$ in your calculator. Then copy the table of values showing the insulin decay pattern at times other than whole-minute intervals.

a.

X	Elapsed Time (min)	0	1.5	4.5	7.5	10.5	13.5	16.5	19.5
Y	Units Remaining in blood	10	9.3	7.9	6.8	5.8	5.0	4.3	3.7

b. Do these points match with the graph of $f(x)=10(0.95)^x$ on the previous page?

Yes!

5. Solve the following equations using the table and/or graph of the function. Estimate to the nearest tenth. Then explain what your answer means.

a. $2=10(0.95^x)$

Solution: $x = 31.4$

Explanation: It takes 31.4 minutes for only 2 units of insulin to remain in the bloodstream.

b. $8=10(0.95^x)$

Solution: $x = 4.4$

Explanation: After 4.4 minutes, there are 8 units of insulin in the bloodstream.

c. $10(0.95^x)>1.6$

Solution: $x < 35.7$

Explanation: There are more than 1.6 units of insulin in the blood until 35.7 minutes after the medicine is taken.

d. $f(49)=10(0.95)^{49}$

Solution: $f(49) \approx 0.8$

Explanation: After 49 minutes, 0.8 units of insulin remain.

6. Give the practical domain (x) of the insulin problem.

Any rational number > 0.

7. Give the practical range (y) of the insulin problem.

Any rational number ≤ 10 but ≥ 0 (between 0 & 10, inclusive)

8. Use the function $f(x) = 10(0.95)^x$ to estimate the half-life of insulin when....

a. The initial dose is 10 units, the half-life is approximately 13.5 minutes.

b. The initial dose is 15 units, the half-life is approximately 13.5 min.

c. The initial dose is 20 units, the half-life is approximately 13.5.

d. The initial dose is 25 units, the half-life is approximately 13.5.

e. The pattern is... the initial amount does not affect the half-life.

9. Car dealerships don't like to keep used cars on their lot for long because most people that buy a new car trade in an old one. In order to keep their inventory of used cars low, they lower the price of the used cars by 3% each month. Use a car that starts off at $12,500 for this problem.

a. What function $p(m)$ will model the price of the car after any number of months (m)?

$p(m) = 12{,}500(\underbrace{0.97}_{\text{\% remaining}})^m$

b. Write a recursive equation that will model this situation.

$$\begin{cases} P_0 = 12500 \\ P_n = P_{n-1} \cdot 0.97 \end{cases}$$

c. What will the price of the car be after 1 month?

$p(1) = 12500(0.97)^1$ $\boxed{\$12{,}125}$

d. What will the price of the car be after 1 year?

→ 12 mo.

$p(12) = 12500(0.97)^{12}$

$\approx \boxed{\$8{,}673.03}$

e. When will the car be worth at most $7000?

$12500(0.97)^m \leq 7000$ Graph & find intersection!

$\boxed{m \geq 19.04 \text{ months}}$

f. When will the price of the car be half of the original price?

$\frac{12500}{2} = 6250$ $6250 = 12500(0.97)^m$ Graph & find intersection

$\boxed{m \approx 22.76 \text{ months}}$

g. What is a realistic domain and range for this situation?

(months)
Domain –
Rational numbers ≥ 0

(Price)
Range –
Rational numbers between 0 & 12,500, inclusive.